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DETERMINATION OF THE HEAT-TRANSFER CHARACTERISTICS IN A CHANNEL OF ANNULAR CROSS SECTION WITH SPIRAL FINS

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The characteristics of heat transfer in the developed turbulent flow of a viscous incompressible liquid in a slot channel of annular cross section with spiral fins are analyzed. Expressions are obtained for calculating the Nusselt numbers at the convex and concave walls of the channel.

In the literature there are experimental data on the asymmetry of the averaged velocity profiles and the distributions of pulsation components in curved channels [1-4]. One can presume the existence of the corresponding asymmetry of the conditions of heat transfer between the heat-transfer agent and the walls in a curved channel, which is also confirmed experimentally for plane curved channels [1].

Let us estimate the possible difference between the values of the heat-transfer coefficient $\alpha_{1,2} = q_{1,2}/(T_{1,2} - \bar{T})$ at the convex and concave walls of a channel of annular cross section with spiral fins as a function of the geometrical characteristics of the channel, the physical characteristics of the heat-transfer agent, and the hydrodynamic parameters of the flow. We carry out the analysis using the methods and assumptions adopted in the investigation of hydraulics and heat transfer in smooth annular channels without fins and plane curved channels [1-6]. We consider the turbulent flow of an incompressible viscous heat-transfer agent in an annular channel with spiral fins under steady-state conditions of moderate heat fluxes and velocities, outside the region of the disturbing action of the fins. We assume that the thermophysical properties of the liquid are constant and the heat flux through the walls of the annular channel is constant along the channel length and angularly.

We assume that secondary flows are absent, i.e.,

$$V_r = 0,\ V_z = V_\varphi \frac{S}{2\pi r},\ \ \tau_z = \tau_\varphi \frac{S}{2\pi r},$$
$$\frac{\partial P}{\partial z} = \frac{\partial P}{r\partial\varphi}\frac{S}{2\pi r},\ \ \frac{1}{r}\frac{\partial V}{\partial\varphi} = -\frac{S}{2\pi r}\frac{\partial V}{\partial z},\ \ \frac{\partial T}{\partial z} = \frac{\partial T}{r\partial\varphi}\frac{S}{2\pi r}. \tag{1}$$

At the inner and outer walls,

$$V_{1,2} = 0,\ q_{1,2} = \alpha_{1,2}(T_{1,2} - \overline{T}) = -\lambda \left.\frac{\partial T}{\partial r}\right|_{r=r_{1,2}}. \tag{2}$$

We determine the characteristics of heat transfer at the inner and outer walls of the channel in the form of the functions

$$\mathrm{Nu}_{1,2} = f(\mathrm{Re},\ \mathrm{Pr},\ S/2\pi r_2,\ \delta/r_2,\ q_1/q_2),$$
$$\mathrm{Nu}_{1,2} = \alpha_{1,2} d_h/\lambda = 2q_{1,2}\delta/\lambda(T_{1,2} - \overline{T}),\ \delta = r_2 - r_1.$$

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In such a statement the temperature profile T(r) can be determined from the heat-transfer equation

$$\rho C_p V_z \frac{\partial T}{\partial z}\left[1+\left(\frac{2\pi r}{S}\right)^2\right]=-\frac{1}{r}\frac{\partial}{\partial r}\left[(\lambda+\lambda_t) r\frac{\partial T}{\partial r}\right]. \qquad (3)$$

Here the functions $V_z(r)$ and $\lambda_t(r) = \lambda[\varepsilon(r)/\nu](Pr/Pr_t)$ are constructed within the framework of a semiempirical four-layer model of the flow, similar to that used in [1] to calculate the velocity profile of an incompressible liquid in a plane circular channel. Integrating (3) over r from r_1 to r_2, we obtain

$$\frac{\partial T}{\partial z}=\frac{r_1 q_1+r_2 q_2}{\rho C_p\int\limits_{r_1}^{r_2} V_z\left[1+\left(\frac{2\pi r}{S}\right)^2\right] r dr}. \qquad (4)$$

We obtain the temperature profile of the heat-transfer agent over r as a result of twofold integration of (3) over r using (2) and (4):

$$T_2-T=\frac{\delta q_2}{\lambda}\int\limits_{\xi}^{1}\frac{1}{[(1-r^*)\xi+r^*]\left(1-\frac{\lambda_t}{\lambda}\right)}\left\{\frac{r^*q^*+1}{\int\limits_0^1\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r^*)\xi+r^*]\,d\xi}\times\right. \qquad (5)$$

$$\left.\times\int\limits_0^{\xi}\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r^*)\xi+r^*]\,d\xi-r^*q^*\right\}d\xi.$$

Here $q^* = q_1/q_2$; $r^* = r_1/r_2$; $\xi = (r - r_1)/(r_2 - r_1)$; $\varphi^* = V_z/\bar{V}_z$; $S^* = S/2\pi r_2$. The temperature of the liquid averaged over the radius is

$$\bar{T}=\frac{2}{\bar{V}_z(r_2^2-r_1^2)}\int\limits_{r_1}^{r_2} TV_z r dr=\frac{2}{1+r^*}\int\limits_0^1 T\varphi^*[(1-r^*)\xi+r^*]\,d\xi.$$

Substituting the function for T from (5) into the latter expression and keeping in mind that

$$\frac{2}{1+r^*}\int\limits_0^1\varphi^*[(1-r^*)\xi+r^*]\,d\xi=1,$$

we obtain the expression for $T_2 - \bar{T}$. Similarly, we determine the value of $T_1 - \bar{T}$ and then $Nu_{1,2} = d_h q_{1,2}/(T_{1,2} - \bar{T})\lambda$:

$$\frac{1}{Nu_1}=\frac{2\delta/d_h}{(1+r^*)q^*}\int\limits_0^1\frac{\int\limits_{\xi}^{1}\varphi^*[(1-r^*)\xi+r^*]\,d\xi}{[(1-r^*)\xi+r^*](1+\lambda_t/\lambda)}\left\{\frac{r^*q^*+1}{\int\limits_0^1\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r^*)\xi+r^*]\,d\xi}\times\right. \qquad (6)$$

$$\left.\times\int\limits_{\xi}^{1}\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r^*)\xi+r^*]\,d\xi-1\right\},$$

$$\frac{1}{Nu_2}=\frac{2\delta/d_h}{1+r^*}\int\limits_0^1\frac{\int\limits_0^{\xi}\varphi^*[(1-r^*)\xi+r^*]\,d\xi}{[(1-r^*)\xi+r^*](1+\lambda_t/\lambda)}\left\{\frac{r^*q^*+1}{\int\limits_0^1\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r_\omega^*)\xi+r_\omega^*]\,d\xi}\times\right. \qquad (7)$$

$$\left.\times\int\limits_0^{\xi}\varphi^*\left\{1+\left[\frac{(1-r^*)\xi+r^*}{S^*}\right]^2\right\}[(1-r^*)\xi+r^*]\,d\xi-r^*q^*\right\}d\xi.$$

The hydraulic diameter of an annular channel with spiral fins is

$$d_h = \frac{4\delta(1+r^*)}{(\sqrt{1+(r^*/S^*)^2}+\sqrt{1+1/S^{*2}})(r^*/\sqrt{1+(r^*/S^*)^2}+1/\sqrt{1+1/S^{*2}})}.$$

From the expressions presented above one can easily obtain simpler functions for the particular variants of one-sided heat supply ($q^* = 0$ and $q^* = \infty$), a plane round channel ($S^* = 0$), a plane straight channel ($r^* \to 1$), and a cylindrical pipe of round cross section ($r^* = 0$, $q^* = 0$, $S^* = \infty$). In the latter case the expression for Nu_2 coincides with the function obtained in [7] for heat exchange in a pipe and satisfactorily confirmed by experimental data.

The function $\varphi^*(\xi)$ appearing in the expressions for $Nu_{1,2}$ is determined using a semi-empirical four-layer model of the flow. According to the data of experimental investigations of the characteristics of turbulent flow in a plane curved channel [2, 3], in the region of the stream near a wall the radial velocity profile is determined by wall friction, as in straight pipe flow. Therefore, we take the law of velocity variation in the two layers of the model closest to the walls by analogy with a plane straight channel:

$$0 \leqslant y^+_{1,2} \leqslant 5,\quad V/V^*_{1,2} = y^+_{1,2},$$

$$5 \leqslant y^+_{1,2} \leqslant 30,\quad V/V^*_{1,2} = -3{,}05 + 5\ln y^+,$$

$$V^*_{1,2} = \sqrt{\frac{\tau_{1,2}}{\rho}},\quad y^+_{1,2} = y_{1,2}V^*/\nu,\quad y_{1,2} = |r - r_{1,2}|.$$

For the developed turbulent flow in the third layer from the wall we use an empirical power function proposed in [2] for a plane curved channel,

$$V/V^*_{1,2} = a\,(y^+_{1,2})^{1/n_{1,2}},$$

where $n_{1,2} = f\left(\dfrac{\nu}{r_{S1,2}\sqrt{\tau_{1,2}/\rho}}\right)$.

In the fourth layer of the model we take the profile $V_\varphi r = \text{const}$ ($V\sqrt{r r_S} = \text{const}$), determined experimentally in a plane circular channel [2] (this profile corresponds to an inversely proportional dependence between the velocity of a liquid element and the length of its path along the trajectory of the averaged motion between two cross sections of P = const):

$$\varphi^*(\xi) = \frac{V_z}{\bar{V}_z} = \frac{V}{V^*_{1,2}}\,\frac{\cos\beta}{\sqrt{\cos\beta_{1,2}}}\,\frac{V^*_z}{\bar{V}_z}.$$

The quantities $\tau_{1,2}$ appearing in the expressions for $V^*_{1,2}$ and $y^+_{1,2}$ and the function $\tau(r)$ required for the calculation of $\varepsilon(r)$ are obtained as a result of a solution of the equation of conservation of momentum [8] in the axial direction

$$\rho\frac{\partial V_z}{\partial t} = \frac{\partial}{\partial z}(\tau^*_{zz} - \rho V_z^2 - \rho\overline{V_z'^2}) + \frac{1}{r}\frac{\partial}{\partial r}[r(\tau^*_{rz} - \rho V_r V_z - \rho\overline{V_r'V_z'})] + \frac{\partial}{r\partial\varphi}(\tau^*_{\varphi z} - \rho V_\varphi V_z - \rho\overline{V_\varphi' V_z'}),$$

where

$$\tau^*_{zz} = -P + 2\mu\frac{\partial V_z}{\partial z};\quad \tau^*_{rz} = \mu\left[\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}\right];\quad \tau^*_{\varphi z} = \mu\left[\frac{\partial V_\varphi}{\partial z} + \frac{1}{r}\frac{\partial V_z}{\partial\varphi}\right].$$

For the case of steady flow, $V_r = 0$, and under the assumption that the pulsation velocity components along φ and z are constant far from the fins, the equation of motion takes the form

$$\frac{\partial P}{\partial z} = 2\mu\frac{\partial^2 V_z}{\partial z^2} - \rho\frac{\partial}{\partial z}(V_z^2) + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial}{r\partial\varphi}\tau_{\varphi z} - \rho\frac{\partial}{r\partial\varphi}(V_\varphi V_z), \qquad (8)$$

where $\tau_{rz} = \tau^*_{rz} - \rho\overline{v_r'v_z'}$; $\tau_{\varphi z} = \tau^*_{\varphi z} - \rho\overline{v_z'v_\varphi'}$.

In accordance with (1),

$$\frac{\partial\tau_{\varphi z}}{r\partial\varphi} = -\frac{S}{2\pi r}\frac{\partial\tau^*_{\varphi z}}{\partial z} = -\frac{S}{2\pi r}\frac{\partial}{\partial z}\left[\mu\frac{\partial V_z}{\partial z}\left(\frac{2\pi r}{S} - \frac{S}{2\pi r}\right)\right] = -\mu\frac{\partial^2 V_z}{\partial z^2}\left[1 - \left(\frac{S}{2\pi r}\right)^2\right];$$

$$\frac{\partial}{r\partial\varphi}(V_\varphi V_z) = \frac{2\pi r}{S}\frac{\partial}{r\partial\varphi}(V_z^2) = -\frac{\partial}{\partial z}(V_z^2),$$

and hence

$$\frac{\partial P}{\partial z} = \mu\frac{\partial^2 V_z}{\partial z^2}\left[1 + \left(\frac{S}{2\pi r}\right)^2\right] + \left(\frac{\partial}{\partial r}\tau_{rz} + \frac{\tau_{rz}}{r}\right). \tag{9}$$

Let us estimate the order of magnitude of the terms on the right side of Eq. (9). As the scale for V_z we take $\bar{V}_z$, for r we take r_2, for dr we take $\delta/2$, and for z we take $S/a = h\sqrt{1 + (S/2\pi r_2)^2}$ (where a is the number of fins; h is the distance between fins on the involute of the outer wall of the channel, which is the analog of the width of a plane channel). The order of τ_{rz} over the entire thickness δ of the channel is no less than the order of $\mu(\partial V_z/\partial r)$ and hence the order of the second term on the right side of (9) is no less than $(4\mu\bar{V}_z/\delta^2)\cdot[1 + (\delta/2r_2)]$. The ratio of the orders of the first and second terms exceeds $(\delta/h)^2/4(1 + \delta/2r_2)$, which is less than 0.01 for $\delta/h = 0.2$. Neglecting the first term on the right side of (9), we obtain a first-order linear differential equation for τ_{rz}, the solution of which gives the function $\tau_{rz}(r)$:

$$\tau_{rz} = \frac{1}{2}\frac{\partial P}{\partial z}\frac{r^2 - R_m^2}{r} = \frac{1}{2}\left(\frac{\partial P}{\partial z}\right)_2 \frac{1 + 1/S^{*2}}{1 + (2\pi r/S)^2}\frac{r^2 - R_m^2}{r}. \tag{10}$$

The quantity $(\partial P/\partial z)_2$ can be expressed through the coefficient of hydraulic resistance ζ_z and the average axial velocity component of the gas:

$$\left(\frac{\partial P}{\partial z}\right)_2 = -\zeta\frac{1}{2\delta}\frac{\rho\bar{V}_z^2}{2}.$$

Using the expression for τ_{rz}, we determine $\tau_{r\varphi} = \tau_{rz}(2\pi r/S)$ and $\tau = \sqrt{\tau^2_{r\varphi} + \tau^2_{rz}}$. The relation for $\tau_{r\varphi}$ can be obtained through the solution of the equation of conservation of angular momentum in the φ direction, analogous to what was done for τ_{rz}. The equation connecting y^+ and ξ includes $\tau_{1,2}$: $y^+ = f(\xi, Re_z, f_z, \delta/r_2, S^*, \xi_m)$. Thus, for the three layers of the four-layer flow model closest to the walls we can write $\varphi^* = f(\xi, Re_z, \xi_z, \delta/r_2, S^*, \xi_m)$. For the fourth layer, where $V_\varphi r$ = const, or in the more convenient form $V_\varphi r = k\bar{V}_\varphi r_2$,

$$\varphi^* = \frac{k(1 + r^*)}{2[(1 - r^*)\xi + r^*]^2}. \tag{11}$$

In deriving the expression for the velocity profile in a spiral channel it was assumed that the radial-average value of the velocity does not vary along φ. Within the framework of the semiempirical applied model under consideration this assumption is fully justified, since the calculated functions $V(\varphi)$ and $V(z)$ far from the fins for the actual parameters of the channel and liquid are extremely weakly expressed. We obtain the function $V(\varphi)$ from the first equation of the system of equations of motion, which under these conditions has the form

$$\frac{\partial P}{\partial r} = \rho\frac{V_\varphi^2}{r},$$

and from the relation, due to the geometry of the channel,

$$\frac{\partial P}{\partial\varphi} = \left(\frac{\partial P}{\partial\varphi}\right)_2\left(\frac{r}{r_2}\right)^2\frac{1 + 1/S^{*2}}{1 + (r/S^*r_2)^2}.$$

Differentiating the first equation with respect to φ, and the second with respect to r, and equating the right sides, we obtain

$$\frac{\partial}{\partial\varphi}(V_\varphi^2) = C_1(r),$$

from which

$$V_\varphi(r, \varphi) = \sqrt{C_1(r)\varphi + C_2},$$

$$C_1 = \frac{2}{\rho}\left(\frac{\partial P}{\partial\varphi}\right)_2\frac{[1 + 1/S^{*2}](r/r_2)^2}{[1 + (r/S^*r_2)^2]^2}.$$

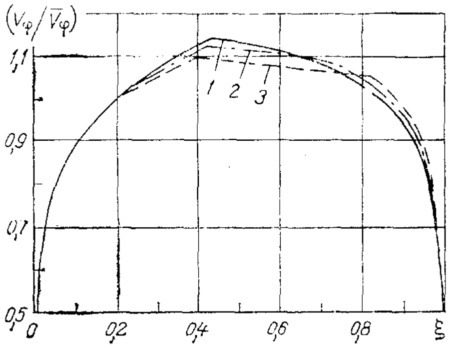

Fig. 1. Calculated velocity profile in a spiral channel with $r^* = 0.9$ and $S^* = 1$: 1) $Re = 5.23 \cdot 10^4$; 2) $1.05 \cdot 10^5$; 3) $5.3 \cdot 10^5$.

We thus obtain the dependence for V(z). For a channel with one spiral fin $S^* = 1$, $\delta/r_2 = 0.9$, and $\zeta = 0.04$ the calculated values of V_φ on the two sides of the fin differ from the value of V_φ averaged over φ by about 2.5%.

The coefficient k in (11) and the coordinates for joining the power-law profiles and the profile $V_\varphi(r) = \text{const}$ are determined from the condition

$$2\int_{r_1}^{r_2} V_z r dr = \bar{V}_z (r_2^2 - r_1^2)$$

and the condition of equality of the velocities at the joining points. The coordinate ξ_m, where $\tau_{rz} = \tau_{r\varphi} = 0$, was calculated from the equation

$$\xi_m = \xi_{m0} + (0{,}5 - \xi_{m0})/10^7 (1 - r_{s1}/r_{s2}),$$

which for $\xi_{m0} = 0.32$ well approximates the experimental data for a plane circular channel presented in [3]. According to these data, in turbulent flow in a plane circular channel ξ_m hardly depends on Re and is a function of r^* only. The function $\varepsilon(\xi)$ is determined from the relation

$$\tau = \sqrt{\tau_z^2 + \tau_\varphi^2} = \rho(\nu + \varepsilon)\sqrt{\left(\frac{\partial V_z}{\partial r}\right)^2 + \left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}\right)^2}.$$

The expression $\tau_\varphi = \rho\nu\left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}\right)$ is adopted for spiral flow by analogy with flow in a plane circular channel, where it describes most accurately the experimental data of [4] on the distribution of τ over a channel cross section. It should be noted that the relations $V_\varphi = V_z(2\pi r/S)$ and $\tau_\varphi = \tau_z(2\pi r/S)$, following from the geometry of spiral flow, are jointly satisfied for $\tau_\varphi = \rho(\nu + \varepsilon)\left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}\right)$ and $\tau_z = \rho(\nu + \varepsilon)\frac{\partial V_z}{\partial r}$, but not under the assumption $\tau_\varphi = \rho(\nu + \varepsilon)\left(\frac{\partial V_\varphi}{\partial r} + \frac{V_\varphi}{r}\right)$ or $\tau_\varphi = \rho(\nu + \varepsilon)\frac{\partial V_\varphi}{\partial r}$, which are also considered in the analysis of flows in curved channels.

The function $Nu_{1,2} = f(Re, Pr, S^*, r^*, q^*)$ and the dimensionless temperature profile $T^* = (T - \bar{T})GC/2\pi\delta(r_1q_1 + r_2q_2)$ were calculated for a plane curved channel ($S^* = 0.01$) and a channel of annular cross section with spiral fins ($S^* = 1$).

In the first stage of the calculation the velocity profile $V/\bar{V} = f(\xi)$ was determined from the given values of Re, r^*, S^*, and ξ_{m0}. Then the function $\varepsilon/\nu = f(\xi)$ was calculated, and the quantities Nu_1 and Nu_2 and the distribution $T^*(\xi)$ were calculated for a number of values of q and Pr/Pr_t. Preliminary calculations of the dimensionless temperature profile $T^*(\xi)$ showed that in the vicinity of ξ_m, where the value of ε/ν calculated from the equations obtained decreases to zero, there is strong distortion of the function T^* (a section appears where $\partial^2T/\partial\xi^2 < 0$ for $q_1 > 0$ and $q_2 > 0$). The experimental data on the distribution

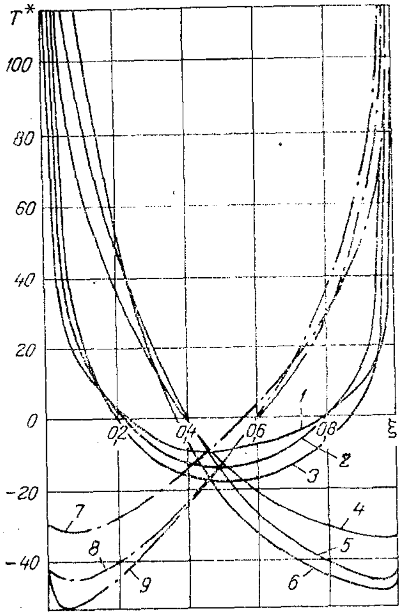

Fig. 2

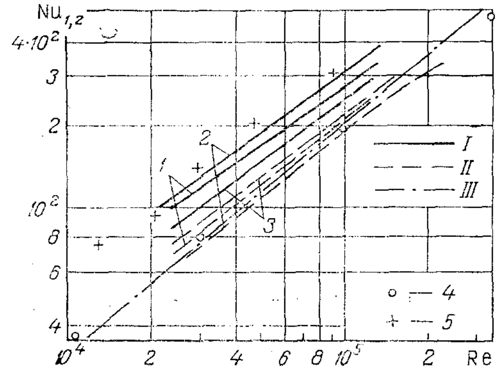

Fig. 3

Fig. 2. Calculated temperature profiles in a spiral channel under conditions of symmetric (a) and asymmetric (b, c) heat supply for $r^* = 0.9$, $S^* = 1$, and $Pr/Pr_t = 1$. a. $q^* = 1$: 1) $Re = 2.5 \cdot 10^4$, $Nu_1 = 81$, $Nu_2 = 93$, $T^*_1 = 112$, $T^*_2 = 98$; 2) $1.05 \cdot 10^5$, 211, 218, 180, 174; 3) $5.3 \cdot 10^5$, 338, 348, 566, 550. b. $q^* = 100$: 4) $Re = 2.5 \cdot 10^4$, $Nu_1 = 71$, $Nu_2 = -6$, $T^*_1 = 276$, $T^*_2 = -34$; 5) $1.05 \cdot 10^5$, 186, −18, 437, −44; 6) $5.3 \cdot 10^5$, 322, −101, 1270, −40. c. $q^* = 0.01$: 7) $Re = 2.5 \cdot 10^4$, $Nu_1 = -6$, $Nu_2 = 79$, $T^*_1 = -31$, $T^*_2 = 221$; 8) $1.05 \cdot 10^5$, −17, 193, −43, 378; 9) $5.3 \cdot 10^5$, −84, 333, −44, 1100.

Fig. 3. Calculated function $Nu_{1,2}(Re)$ for a spiral channel under conditions of symmetric heat supply $q^* = 1$ for $Pr/Pr_t = 1$: I) Nu_2; II) Nu_1; III) $0.021 Re^{0.8}$; 1) $r^* = 0.9$, $S^* = 0.01$; 2) 0.8, 0.01; 3) 0.9, 1; 4) 0.5, ∞ [5]; 5) 0, 1 [6].

of ε/ν over the cross section of a curved channel [2] and of a channel of annular cross section [5] do not give an exact representation of the value of ε/ν near ξ_m. In any case, the data of [5] indicate that the value of ε/ν at $\xi = \xi_m$ does not equal zero in a channel of annular cross section. Therefore, the final calculations of the profile $T^* = f(\xi)$ were made with a ratio ε/ν varying linearly in the vicinity of ξ_m, analogous to what was done in [1] in calculating Nu for a plane curved channel ("bridge scheme" [9]).

The results of the calculations are presented in Figs. 1-4.

Each of the graphs of $Nu_1(q^*)$ (Fig. 4) has a discontinuity ($+\infty$, $-\infty$) at a certain value of $q^* < 1$, while $Nu_2(q^*)$ has one at $q^* > 1$. The sharp increase in Nu_1 and Nu_2 as this value of q^* is approached does not signify an improvement of the heat transfer in the gas, but indicates a decrease in the difference $T_{1,2} - \bar{T}$, when the considerable difference between q_1 and q_2 results in an essentially asymmetric temperature field along r, and the quantity $\bar{T}$ becomes as close as desired to the temperature of the colder wall and can be either greater or less than it. When the usual function Nu(Re, Pr) for symmetric heat supply is applied to the problem of asymmetric heat supply, it is possible to overstate the calculated temperature of the adiabatic wall or the wall through which the lesser heat flux is supplied to the heat-transfer agent (if the wall temperature is defined as $T_{1,2} = \bar{T} + q_{1,2}/\alpha_{1,2}$).

An analysis of the dependence of $Nu_{1,2}$ on Re shows that for $Pr/Pr_t = 1$ the values of Nu_1 and Nu_2 are proportional to Re^{a}, where $a \approx 0.8$ for $Re < 5 \cdot 10^5$; the exponent a decreases with an increase in Re.

The dependence of $Nu_{1,2}$ on the Prandtl number for $Re < 10^5$ can be represented in the form $Nu_{1,2} = kPr^b$, where $k = f(Re, S^*, r^*)$, while the exponent b (for $0.3 < b < 0.5$) is larger for Nu_1 than for Nu_2 and increases with an increase in Re.

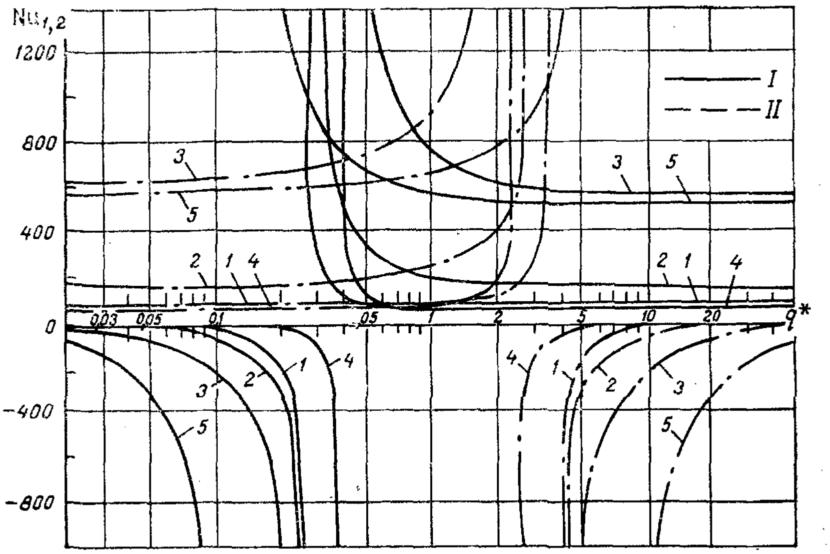

Fig. 4. Calculated function $Nu_{1,2}(q^*)$ for a spiral channel with $r^* = 0.9$, $S^* = 1$, and $Pr/Pr_t = 1$: 1) $Re = 2.5 \cdot 10^4$; 2) $1.05 \cdot 10^5$; 3) $3.5 \cdot 10^5$; $Pr/Pr_t = 0.1$: 4) $Re = 1.05 \cdot 10^5$; $Pr/Pr_t = 10$: 5) $Re = 1.05 \cdot 10^5$; I) Nu_1; II) Nu_2.

It should be noted that for all the values of Re, Pr/Pr_t, r^*, S^*, and $q^* = 1$ considered, the Nusselt number Nu_2 at the outer concave wall of the channel exceeds the value of Nu_1 at the convex inner wall. These calculated results are in qualitative agreement with the data of an analysis of the stability of flow near convex and concave walls [2] and with the data of measurements of the pulsation components of the gas velocity in a plane curved channel [4], indicating the suppression of radial pulsations near the inner convex wall of the channel and their strengthening near the outer concave wall.

In the vicinity of $Pr/Pr_t = 1$ for all the values of the Reynolds number considered ($2.4 \cdot 10^4$-$5.3 \cdot 10^4$) the ratio Nu_2/Nu_1 is larger in the case of $r^* = 0.8$ than for $r^* = 0.9$ (Fig. 4), Nu_2 is larger for $r^* = 0.8$ than for $r^* = 0.9$, while Nu_1 is correspondingly smaller. For $r^* = 0.9$ the ratio Nu_2/Nu_1 is considerably larger in the case of a plane curved channel ($S/2\pi r_2 = 0.01$) than in an annular channel with spiral fins.

Thus, as a result of the analysis made under the adopted assumptions, we can conclude that in the analyzed range of geometrical parameters ($r^* = 0.8$-0.9, $S^* = 0.01$-1) the characteristics of the heat transfer at the outer (concave) wall of an annular channel with spiral fins are improved, while at the inner (convex) wall they are worsened in comparison with flow in a straight channel with a decrease in r_1/r_2 and $S/2\pi r_2$. In the particular asymptotic cases as $r^* \to 1$ and $S^* \to \infty$ the calculated function $Nu_{1,2}(Re, Pr)$ approaches the well-known empirical function $Nu = kRe^{0.8}Pr^{0.43}$.

In presuming the existence of a unique dependence between the radius of curvature of the trajectory of the averaged motion of elements of the liquid near the convex and concave walls and the weakening and intensification of heat transfer to the liquid near the respective walls, one must keep in mind the character of the dependence of the radius of curvature of the spiral trajectory on the radial coordinate: $r_s = r[1 + (S/2\pi r)^2]$. Here r_s has a minimum value at $r = S/2\pi$ (the angle of inclination of the fins to the axis is $\beta = 45°$), i.e., variants of an annular channel with spiral fins are possible where: a) $\partial r_s/\partial r > 0$ for $r_1 < r < r_2$ ($S/2\pi r_1 \geq 1$); b) $\partial r_s/\partial r < 0$ for $r_1 < r < r_2$ ($S/2\pi r_2 \leq 1$); c) $\partial r_s/\partial r < 0$ for $r_1 < r < r_0$ and $\partial r_s/\partial r > 0$ for $r_0 < r < r_2$ ($S/2\pi r_1 < 1$ and $S/2\pi r_2 > 1$). For the maximum intensification of heat transfer to the concave wall of a spiral channel it is evidently advisable to have $S = 2\pi r_2$ ($\beta_2 = 45°$).

NOTATION

r, φ, z, radial, angular, and axial coordinates, respectively; l, n, curvilinear coordinates directed along the spiral trajectory and perpendicular to it and r, respectively; V, velocity; v', pulsation component of velocity; P, pressure; τ, shear stress; q, specific heat flux; T, temperature; α, coefficient of heat transfer to the channel wall; λ, coefficient of thermal conductivity of the liquid; ρ, density; C, heat capacity; d_h, hydraulic diameter of the channel; S, pitch of the spiral fins; Nu, Nusselt number; Re, Reynolds number; Pr, Prandtl number; Pr_t, turbulent Prandtl number; ε, coefficient of turbulent transfer of momentum; ν, kinematic viscosity; μ, dynamic viscosity; R_m, radial coordinate of the surface where $\tau = 0$; r_s, radius of curvature of the spiral trajectory; G, mass flow rate of the liquid; β, angle of inclination of the spiral trajectory of an element of liquid to the z axis at the radius r. Indices: 1, 2, conditions at the inner (convex) and outer (concave) walls of the channel, respectively.

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INFLUENCE OF SOLID DEPOSITS ON THE INCEPTION OF SELF-EXCITED THERMOACOUSTIC OSCILLATIONS IN HEAT TRANSFER TO TURBULENT FLUID FLOW IN TUBES

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It is established experimentally that solid carbon deposits formed in heat transfer to kerosene in small-bore tubes induce self-excited thermoacoustic oscillations.

Perspectives have changed considerably in recent years in regard to the nature of solid deposits formed in heat transfer to a fluid. It was previously thought that they merely created an additional heat resistance and caused the temperature of the heat-transfer surface to rise accordingly. It has now been established that the physicochemical and hydrodynamic processes occurring in solid deposits can exert an appreciable influence on the various heat-transfer characteristics and can, depending on their nature and the regime parameters, degrade or improve heat transfer, alter the flow resistance, intensify self-excited thermoacoustic oscillations (STAO), decrease the velocity of sound propagation along the fluid flow, etc. [1-4].

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